# Success of collinear expansion in the calculation of induced gluon emission

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**Abstract**

We clarify the confusion in a recent paper by Aurenche, Zakharov and Zaraket (AZZ) [1] over the procedure and region of validity of the collinear approximation in the twist expansion approach to induced gluon emission in deeply inelastic scattering (DIS) off a nucleus target. We point out that, in this approach to the semi-inclusive spectrum, the transverse momentum $\vec{\ell}_\perp$ of the induced gluon must be fixed in the collinear expansion in the transverse momentum $\vec{k}_\perp$ of the initial partons, therefore the result is valid for $\langle k_\perp^2\rangle \ll \ell_\perp^2 \ll Q^2$. In the twist-four contribution, one can single out the double-hard term corresponding to collinear quark-gluon Compton scattering which can be calculated independently of the collinear expansion approach. We will discuss the connection between the collinear approximation in the twist expansion approach and the small $k_T$ approximation of the results in the Light-Cone Path Integral (LCPI) and Gyulassy-Levai-Vitev (GLV) opacity expansion approach. We point out the misconstrued variable change by AZZ before the $k_T$ expansion in LCPI and opacity expansion approach, without which one obtains the same result for the induced gluon spectrum under collinear approximation as in the twist expansion approach. We also show that corrections beyond the collinear approximation to the transverse momentum integrated gluon spectrum within the static potential model in the GLV approach give rise to a logarithmic factor difference from the result of collinear approximation.

## 1 Introduction

Jet quenching due to parton energy loss in dense medium provides an excellent probe of the hot quark matter produced in high-energy heavy-ion collisions [2]. Such predicted phenomenon was indeed observed in experiments at the Relativistic Heavy-ion Collider (RHIC) [3,4]. Phenomenological studies of the observed jet quenching depend crucially on our theoretical understanding of parton propagation and energy loss in dense medium which is dominated by induced gluon radiation via multiple scattering. There have been, therefore, a

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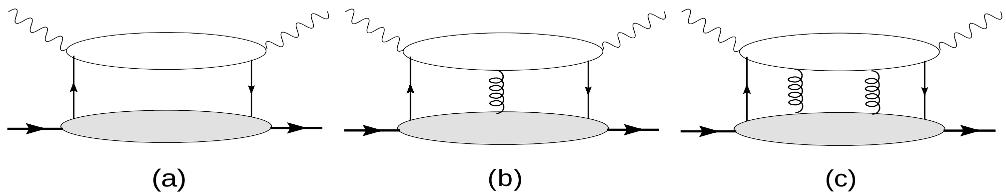

Fig. 1. Diagrams of DIS with multiple gluon interaction

plethora of theoretical studies of induced gluon emission from a propagating parton [5,6,7,8,9,10]. One approach to the study of induced gluon emission is based on higher-twist expansion of multiple parton scattering cross section in a nuclear medium in the framework of collinear factorization approximation [10,11] which can also be applied to parton propagation in hot medium. The advantage of higher-twist approach is the natural formulation of the parton propagation problem in terms of medium modified parton fragmentation function which is the only physical observable of the jet quenching as a result of induced gluon emission and parton energy loss.

The higher-twist approach to the problem of induced gluon emission is so far limited to twist-four contributions in the twist expansion. This is equivalent to the leading order approximation in the opacity expansion approach by Gyulassy, Levai and Vitev (GLV) [9]. The induced gluon spectra from these two approaches can be shown to be equivalent under the twist and opacity expansion approximations [12]. However, Aurenche, Zakharov and Zaraket (AZZ) claim in a recent note [1] that the higher-twist approach "fails" to produce the correct gluon spectra as in the Light-cone Path Integral (LCPI) and opacity expansion approach by Gyulassy, Levai and Vitev (GLV). We want to demonstrate that such an unjustified claim comes from their confusion over the procedure and validity of the collinear expansion in the higher-twist approach. A misconstrued variable change in LCPI and GLV results also lead to their conclusion on the collinear approximation in the $k_T$ factorized formulation. We demonstrate that these results under the same approximation are equivalent to each other.

For semi-inclusive cross section of induced gluon emission, the collinear expansion in terms of the initial parton transverse momentum $k_T$ must be made for a fixed value $\langle k_T^2 \rangle \ll \ell_T^2 \ll Q^2$ of the gluon's transverse momentum. For the total parton energy loss which involves integration over the gluon's transverse momentum, we will argue that the higher-twist expansion is a good approximation for large initial jet energy $q^- \gg R_A \langle k_T^2 \rangle$, where $R_A$ is the nuclear size.

## 2 Collinear factorization and twist expansion

In the twist expansion approach to multiple parton scattering, one considers interaction between a fast parton and the target nucleus (or hot medium) via exchange of soft gluons as shown in Fig. 1 for the case of deeply inelastic scattering (DIS) off a large nucleus. In the framework of collinear factorization, one normally chooses a covariant gauge and considers $A^+$ as the largest component of the gauge field. Since the longitudinal momentum per target nucleon $p = [p^+, 0, \vec{0}_\perp]$ is the largest momentum component in the process, the twist expansion procedure involves expanding the hard part $H_n(k_T)$ of the quark-$n$-gluon interaction in the transverse momentum $k_T$ of the initial gluon fields and the transversely polarized gluon fields $A_\perp$. Using generalized Ward identities one can relate the derivatives of the hard parts $\partial^n_{k_T} H(k_T)_{k_T=0}$ of quark-longitudinal -gluon interaction and the collinear hard parts of quark-transverse-gluon interaction, and combine them to produce gauge invariant higher-twist contributions to the DIS cross section (see Ref. [13] for a detailed illustration). After integration over the transverse momentum of initial gluon fields, the final results are given by the convolution of collinear parton scattering cross sections and transverse momentum integrated parton distribution or correlation matrix elements. The leading twist-four and nuclear enhanced contributions to the hadronic tensor of DIS off a nucleus involve two longitudinal gluon fields [14] as shown in Fig. 1c and can be expressed as

$$W_2 = \int \frac{dy^-}{2\pi} dy_1^- dy_2^- \frac{d^2\xi_T}{(2\pi)^2} d^2k_T e^{-i\vec{k}_T\cdot\vec{\xi}_T} H_2^{--}(k_T) \\ \times \langle A \mid \bar{\psi}(0) \frac{\gamma^+}{2} A^+(y_1^-, 0_T) A^+(y_2^-, \xi_T)\psi(y^-) \mid A\rangle, \quad (1)$$

where we have suppressed the Lorentz indices of the electromagnetic currents and other kinematic variables in the perturbative hard part $H_2^{--}(k_T)$ of the multiple parton scattering with longitudinal gluon fields. Summations over color indices of the field operators in the matrix and average over the color indices of the initial state partons in the hard part are understood. In the collinear factorization scheme, one makes a collinear expansion of the hard part

$$H_2^{--}(k_T) = H_2^{--}(0) + k_T^i \frac{H_2^{--}(k_T)}{\partial k_T^i} \mid_{k_T=0} + \frac{k_T^i k_T^j}{2} \frac{\partial^2 H_2^{--}(k_T)}{\partial k_T^i \partial k_T^j} \mid_{k_T=0} + \cdots . (2)$$

In order to clarify the confusion in Ref. [1] over the collinear expansion, it is important to emphasize here that the longitudinal gauge field $A^+$ is not a physical gluon field. Therefore, the hard part $H_2^{--}(k_T)$ does not correspond to quark interaction with physical gluons. This is apparent in the fact that

the collinear term in the above expansion does not vanish and is actually related to the hard part with no longitudinal gluon interaction (Fig. 1a), $H_2^{--}(0) = (-ig)^2 H_0$. One can prove in general this is true for quark interaction with any number of longitudinal gluons. After integration over the initial parton transverse momentum, which is another important part of the collinear factorization scheme, their contributions to the semi-inclusive cross section take the form

$$H_0 \langle A \mid \bar{\psi}(0)\gamma^+ \left[ 1 - ig \int_0^{y^-} dy_1^- A^+(y_1^-) \right.$$
$$\left. + (-ig)^2 \int_0^{y^-} dy_1^- \int_0^{y_1^-} dy_2^- A^+(y_1^-) A^+(y_2^-) + \cdots \right] \psi(y^-) \mid A \rangle, \tag{3}$$

which becomes part of the leading twist contribution as the gauge link in the gauge invariant quark distribution function

$$f_A^q(x) = \frac{1}{2} \int dy^- e^{ixp^+y^-} \langle A \mid \bar{\psi}(0)\gamma^+ \mathrm{P} e^{-ig\int_0^{y^-} d\xi^- A^+(\xi^-)} \psi(y^-) \mid A \rangle \, . \tag{4}$$

The interaction between a propagating quark and soft longitudinal gauge fields will only result in an eikonal line along the light-cone. This does not correspond to any physical scattering because one can get rid of it by choosing a proper (physical) gauge. This is the basic idea behind the proof of collinear factorization of the leading twist cross section of DIS and Drell-Yan processes by Collins, Soper and Sterman [16].

The contribution from the linear term in Eq. (2) of the collinear expansion vanishes for unpolarized targets. For the quadratic term of the expansion one can combine $k_T^2$ with the longitudinal gauge fields $A^+A^+$ and obtain a quark-gluon correlation distribution after partial integration over $k_T$,

$$\begin{aligned}
&\int \frac{d^2\xi_T}{(2\pi)^2} \frac{dy^-}{2\pi} dy_1^- dy_2^- d^2k_T e^{ixp^+y^- + ix_g p^+(y_1^- - y_2^-) - i\vec{k}_T\cdot\vec{\xi}_T} \\
&\qquad\qquad \times \langle A | \bar{\psi}(0) \frac{\gamma^+}{2} k_T^2 A^+(y_1^-, 0_T) A^+(y_2^-, \xi_T) \psi(y^-) | A \rangle \\
&\approx \int \frac{dy^-}{2\pi} dy_1^- dy_2^- e^{ixp^+y^- + ix_g p^+(y_1^- - y_2^-)} \langle A | \bar{\psi}(0) \frac{\gamma^+}{2} F^{+i}(y_1^-) F_i^+(y_2^-) \psi(y^-) | A \rangle \\
&\approx \pi \int dy_N^- \rho_A(y_N) f_A^q(x) x_g G_N(x_g),
\end{aligned} \tag{5}$$

where a factorized form of the quark-gluon correlation is assumed, $\rho_A(y_N)$ is the nucleon density distribution and $G_N(x_g)$ the gluon distribution func-

tion per nucleon inside the nucleus. The momentum fraction $x_g$ carried by the initial gluon is determined by the kinematics of each individual process. Therefore, the leading twist-four contribution from the quadratic term of the collinear expansion to the semi-inclusive DIS cross section has a simple form

$$\pi \int dy_N^- \rho_A(y_N) \frac{1}{4} \frac{\partial^2 H_2^{--}(k_T)}{\partial^2 k_T} \,|_{k_T=0}, f_A^q(x) x_g G_N(x_g) \tag{6}$$

which has a simple and intuitive interpretation of partonic scattering between the fast quark and a *physical collinear* gluon since the contribution is proportional to the $k_T$-integrated gluon distribution inside the nucleus. The quadratic derivative term $\partial^2_{k_T} H_2^{--}(k_T)_{k_T=0}$ therefore corresponds to the hard part of the actual physical collinear quark-gluon scattering cross section.

If one works in the physical gauge ($A^+ = 0$), the gauge link along the light-cone disappears (there will be transverse gauge link instead, see Ref. [17]). One therefore only has to consider quark interaction with the physical (transverse) gluons. In this case, the hard part $H_2^{\perp\perp}(k_T)$, which will be convoluted with the (physical) gluon distribution, corresponds to partonic cross section of the physical quark-gluon scattering and is completely different from the hard part $H_2^{--}(k_T)$ of quark interaction with longitudinal gluons in the covariant gauge. One can in fact prove in general the equivalence between the collinear hard part $H_2^{\perp\perp}(0)$ in the physical gauge and the second derivative $\partial^2_{k_T} H_2^{--}(k_T)_{k_T=0}$ of the hard part in the covariant gauge [15]. We will illustrate this later in the case of induced gluon radiation.

## 3 Induced gluon spectra in higher-twist expansion

One can calculate nuclear modification to the dijet cross section in DIS [19,18], differential direct photon [20] and Drell-Yan (DY) cross section [21,22] in $p+A$ collisions. The technique of higher-twist expansion has been also been applied to calculate the induced gluon spectrum due to multiple parton scattering in the DIS off a large nucleus or higher-twist contribution to gluon radiative correction to the semi-inclusive DIS cross section in Refs. [10,11]. In this case, one should keep the final gluon transverse momentum $\ell_T$ *fixed* in the collinear expansion of the hard part in the initial parton transverse momentum $k_T$. This is the origin of the flaw that leads to AZZ's conclusion in Ref. [1] about higher-twist approach to induced gluon spectrum.

The dominant contribution in the twist expansion approach mainly comes from the process in Fig. 2. The hard part of the contribution to the semi-inclusive cross section before the collinear expansion is (Eq. (A.18) in Ref. [11]),

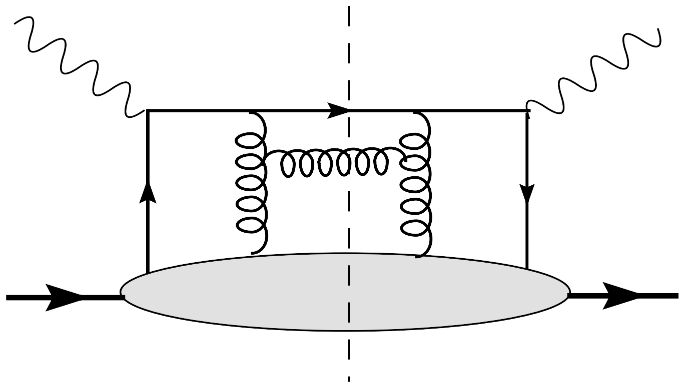

Fig. 2. Diagrams of induced gluon emission in DIS

$$H_2^{--}(k_T) = \frac{1}{(\vec{\ell}_T - \vec{k}_T)^2} \frac{\alpha_s}{2\pi} C_A \frac{1+(1-z)^2}{z} \frac{2\pi\alpha_s}{N_c} e^{ixp^+y^- + i(x_D+x_L)p^+(y_1^- - y_2^-)}$$
$$\times [e^{i(x_D/z+x_L)p^+y_2^-} - 1][e^{i(x_D/z+x_L)p^+(y^- - y_1^-)} - 1], \qquad (7)$$

where

$$x_L = \frac{\ell_T^2}{2p^+q^-z(1-z)}, \qquad x_D = \frac{k_T^2 - 2\vec{k}_T\cdot\vec{\ell}_T}{2p^+q^-(1-z)}, \qquad (8)$$

$$x_D/z + x_L = \frac{(\vec{\ell}_T - \vec{k}_T)^2}{2p^+q^-z(1-z)}. \qquad (9)$$

Note that the overall phase factor quoted in Ref. [1] is different from $\exp[ixp^+y^- + i(x_D+x_L)p^+(y_1^- - y_2^-)]$ of the actual result. However, this does not affect the following argument.

First, one notices that the collinear limit of the hard part $H_2^{--}(0)$ is finite and is the same as the vacuum gluon bremsstrahlung except the phase factors. It combines with the collinear limits of hard parts from other cut diagrams to form the gauge link in the quark distribution function in the vacuum gluon (leading twist) bremsstrahlung process in semi-inclusive DIS. To calculate higher-twist contributions in the twist expansion approach, one keeps the second order in the collinear expansion of the above hard parts in $k_T$ for fixed value of the gluon transverse momentum $\ell_T$. Therefore, one needs only to evaluate the second derivative of the above hard part, $\partial^2_{k_T} H_2^{--}$. The dominant contribution comes from differentiating the factor $1/(\vec{\ell}_T - \vec{k}_T)^2$. As explained in the previous section, one combines the quadratic term $k_T^2$ and the longitudinal gluon fields $A^+A^+$ to form the quark-gluon correlation function after partial integration. The corresponding contribution to the gluon radiation spectrum is

$$
\frac{dN_2}{dz d\ell_T^2} = \frac{\alpha_s}{2\pi} C_A \frac{1+(1-z)^2}{z} \frac{1}{\ell_T^4} \frac{2\pi\alpha_s}{N_c} \frac{T_{qg}^A(x,x_L)}{f_A^q(x)}, \tag{10}
$$

where

$$
\begin{aligned}
T_{qg}^A(x,x_L) = &\int \frac{dy^-}{2\pi}\, dy_1^- dy_2^- (1-e^{-ix_Lp^+y_2^-})(1-e^{-ix_Lp^+(y^- - y_1^-)}) e^{i(x+x_L)p^+y^-} \\
&\times \theta(-y_2^-)\theta(y^- - y_1^-) \frac{1}{2} \langle A | \bar{\psi}_q(0)\, \gamma^+\, F_\sigma^{\ +}(y_2^-)\, F^{+\sigma}(y_1^-)\, \psi_q(y^-) | A\rangle, \quad (11)
\end{aligned}
$$

is the quark-gluon correlation function of the nucleus and $f_A^q(x)$ is the nuclear quark distribution function.

The leading contribution to the second-order derivative $\partial_{k_T}^2 H_2^{--}$ from the phase factor in Eq. (7) will be linear in $(y_1^- - y_2^-)/q^-$ or $y^-/q^-$ which in general are suppressed by a factor $\ell_T^2 r_N/q^-$ ($r_N$ is the nucleon size) or $\ell_T^2/x_B p^+ q^-$ ($x_B$ is fractional momentum of the initial quark) relative to the above leading contribution for large jet energy $q^-$. There are many other power-suppressed terms like these from other diagrams. They can be neglected for $\ell_T^2 \ll Q^2$.

Eq. (10) has both hard-soft, double-hard scattering and their interferences. The double hard scattering is characterized by the finite momentum fraction $x_L$ carried by the initial gluon while in hard-soft contributions the initial gluon carries zero fractional momentum. This result is consistent with that in Ref [18] for nuclear enhancement of jet photoproduction where they consider only hard-soft and double hard scattering, but not their interferences which is not important for large values of $\ell_T^2$.

The higher-twist result also has a simple and intuitive partonic interpretation. One can assume the quark-gluon correlation has a factorized form (see Sec. II of Ref. [23] for details),

$$
\begin{aligned}
T_{qg}^A(x,x_L) = A\pi &\int dy_N^- \rho_A(y_N)\, [f_N^q(x+x_L)[xG_N(x)]_{x=0} + f_N^q(x) x_L G_N(x_L)] \\
&\times [1-\cos(x_Lp^+y_N^-)], \qquad (12)
\end{aligned}
$$

where $f_N^q(x)$ is the quark and $G_N(x)$ the gluon distribution per nucleon, $\rho_A(y_N)$ is the nucleon density at location $y_N$ inside the nucleus $A$.

The second term in the above factorized quark-gluon correlation corresponds to contribution from double hard scattering in which the initial gluon carries finite momentum fraction $x_L p^+$. The corresponding differential higher-twist gluon spectrum is then

$$\frac{dN_2^{(H)}}{dz d\ell_T^2} = \int dy_N^- \rho_A(y_N) \frac{\pi\alpha_s^2}{\ell_T^4} \frac{C_A}{N_c} \frac{1+(1-z)^2}{z} x_L G_N(x_L)[1-\cos(x_L p^+ y_N^-)]$$
$$\equiv \int dy_N^- \rho_A(y_N) \frac{d\sigma_{qg}^N}{dz d\ell_T^2}[1-\cos(x_L p^+ y_N^-)] \tag{13}$$

which can be intuitively interpreted as the differential number of quark-gluon scattering in a collinear factorized form as the quark propagates inside the nucleus, where $d\sigma_{qg}^N/dz d\ell_T^2$ is the collinear quark-gluon cross section on a nucleon target.

One can derive the above double hard scattering contribution from the simple collinear factorized parton model by noting that Fig. 2 in this case is just the quark-gluon Compton scattering process. Considering a quark with momentum $q^-$ scattering with a gluon that carries a fractional momentum $xp^+$, $q(q) + g(xp) \to d(p') + g(\ell)$, the cross section can be written as

$$d\sigma_{ab} = \frac{g^4}{2\hat{s}} |M|^2_{ab\to cd}(\hat{t}/\hat{s}, \hat{u}/\hat{s}) \frac{d^3\ell}{(2\pi)^3 2\ell_0} 2\pi\delta[(xp+q-\ell)^2]$$
$$= \frac{g^4}{(4\pi)^2} |M|^2_{ab\to cd}(\hat{t}/\hat{s}, \hat{u}/\hat{s}) \frac{\pi}{\hat{s}^2} \frac{dz}{z(1-z)} d\ell_T^2 \, \delta\left(1-\frac{x_L}{x}\right), \tag{14}$$

where $q = [0, q^-, 0]$ and $xp = [xp^+, 0, 0]$ are momenta of the initial partons and

$$\ell = \left[\frac{\ell_T^2}{2zq^-}, zq^-, \vec{\ell}_T\right] \tag{15}$$

is the momentum of the final gluon. With the given kinematics, the on-shell condition in the cross section can be recast as

$$(xp+q-\ell)^2 = 2(1-z)xp^+q^-\left(1-\frac{x_L}{x}\right), \quad x_L = \frac{\ell_T^2}{2z(1-z)p^+q^-}. \tag{16}$$

The Mandelstam variables of the collision are,

$$\hat{s} = (q+xp)^2 = 2xp^+q^- = \frac{\ell_T^2}{z(1-z)}, \quad \hat{t} = (\ell - xp)^2 = -z\hat{s},$$
$$\hat{u} = (\ell - q)^2 = -(1-z)\frac{x_L}{x}\,\hat{s} = -(1-z)\hat{s}, \tag{17}$$

where the on-shell condition $x = x_L$ is used.

With Eq. (14) and gluon distribution functions $G_N(x)$, one can obtain the quark-gluon scattering contribution to the quark-nucleon cross section,

$$
\begin{aligned}
d\sigma_{qg}^{N} &= \int d\sigma_{qg} G_N(x) dx \\
&= x_L G_N(x_L) |M|^2_{qg\to qg}(\hat{t}/\hat{s}, \hat{u}/\hat{s}) \frac{\pi\alpha_s^2}{\hat{s}^2} \frac{dz}{z(1-z)} d\ell_T^2
\end{aligned} \tag{18}
$$

Using the quark-gluon scattering matrix element

$$
\begin{aligned}
|M|^2_{qg\to qg} &= \frac{C_A}{N_c}\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} - \frac{C_F}{N_c}\frac{\hat{s}^2+\hat{u}^2}{\hat{u}\hat{s}} \\
&= \left[\frac{C_A}{N_c}\frac{1+(1-z)^2}{z^2} + \frac{C_F}{N_c}\frac{(1+(1-z)^2)}{1-z}\right],
\end{aligned} \tag{19}
$$

the quark-gluon cross section on a nucleon target is

$$
d\sigma_{qg}^{N} = x_L G_N(x_L) \frac{\pi\alpha_s^2}{\ell_T^4}\left[\frac{C_A}{N_c}(1-z) + \frac{C_F}{N_c} z^2\right] \frac{1+(1-z)^2}{z} dz d\ell_T^2, \tag{20}
$$

which is equivalent to the result in Eq. (13) in the soft limit ($z \to 0$).

The divergent factor $1/\ell_T^4$ in the induced gluon spectrum due to double hard scattering arises because of the collinear approximation in which we neglected the transverse momentum of the initial partons. This is related to the neglect of the $k_T$ dependence of the phase factors in the hard parts in the collinear expansions and other higher-twist (larger than four) contributions. These approximations are no longer valid at small values of $\ell_T^2 \ll \langle k_T^2 \rangle$ in the twist expansion approach, since many other contributions and processes will become important which are neglected in the above result. We have argued [10] that contributions of these neglected terms could be approximated by substituting $\ell_T^4 \to 1/\ell_T^2(\ell_T^2 + \mu^2)$ and $x_L G_N(x_L) \to (x_L + x_\mu) G_N(x_L + x_\mu)$ in the collinear result with $x_\mu = \mu^2/2p^+q^-$ and $\mu$ is the average transverse momentum of the medium gluon. However, the interference between double hard and soft rescattering processes suppresses the induced spectra for small $\ell_\perp^2 R_A/2q^- \ll 1$. Therefore, the final result in the collinear expansion will be a good approximation and insensitive to the regularization for large initial quark energy $q^- \gg \hat{R}_A \langle k_T^2 \rangle$. For corrections beyond the twist-four contribution, one has to consider the nuclear broadening of the transverse momentum $\langle k_T^2 \rangle \sim R_A \hat{q}$ ($\hat{q}$ is the jet transport parameter [23])

The double hard scattering process corresponds to elastic scattering in which there is a finite energy transfer ($x_L$) from the medium gluon. With the leading order contribution to the medium gluon distribution $xG_N(x) \sim \delta(x-1)$, the corresponding energy loss can be proved to be the same as the elastic energy loss [24]. This is a unique feature of the twist expansion approach that is not included in all other approaches (BDMPS and GLV). On the other hand, the

quantum non-locality of the quark-photon interaction in DIS has never been considered as an important feature of the twist expansion approach.

One can similarly interpret the first term in Eq. (11) which corresponds to the hard-soft process. Here the gluon radiation is induced by the hard photon-quark scattering and subsequently the radiated gluon interacts with a soft gluon from another nucleon with distribution $xG_N(x)_{x=0}$. One can compare this part of gluon spectrum

$$\frac{dN_2^{(S)}}{dzd\ell_T^2} = \int dy_N^- \rho_A(y_N) \frac{\pi\alpha_s^2}{\ell_T^4} \frac{C_A}{N_c} \frac{1+(1-z)^2}{z} xG_N(x)_{x\approx 0} \times [1 - \cos(x_L p^+ y_N^-)] \quad (21)$$

to the induced gluon spectra in LCPI and GLV approaches, especially after substitution $xG_N(x)_{x\approx 0} \to x_\mu G_N(x_\mu)$ and $\ell_T^4 \to 1/\ell_T^2(\ell_T^2 + \mu^2)$ when the effect of the finite transverse momentum of the initial gluon is considered.

Note that with higher order contributions to the medium gluon distribution function $xG_N(x)$, the semi-inclusive spectrum from the double hard scattering is similar to the hard-soft scattering with a correction on the order of $(\ell_T^2/Q^2)[x\partial_x G_N(x)]_{x\approx 0}$ which can be neglected for small values of $\ell_T^2 \ll Q^2$.

## 4 Induced gluon emission in $k_T$-factorized form

The differential spectrum for induced gluon emission via interaction between the fast quark and the medium or initial gluon in the LCPI formulation is obtained in the $k_T$ factorized form,

$$\frac{dN_{\rm LCPI}}{dzd\ell_T^2} = \frac{1+(1-z)^2}{z} \int dy_N^- \rho_A(y_N) \int d^2k_T \frac{xdG_N(k_T^2, x)}{d\ln k_T^2} \widetilde{H}(\vec{k}_T),$$
$$\widetilde{H}(\vec{k}_T) = 2\pi\alpha_s^2 \frac{\vec{k}_T \cdot \vec{\ell}_T}{\ell_T^2(\vec{k}_T - \vec{\ell}_T)^2} \left[1 - \cos\frac{(\vec{\ell}_T - \vec{k}_T)^2 y_N^-}{2q^- z(1-z)}\right], \quad (22)$$

where $q^-$ is the energy of the fast quark, $k_T$ the transverse momentum of the medium or initial gluon, $\ell_T$ the transverse momentum of the emitted gluon (see Ref. [25,26] for similar formula in the GLV approach). Note that in Eqs. (12)-(14) in Ref. [1] the transverse momentum of the emitted gluon is replaced by $\vec{\ell}_T \to \vec{\ell}_T - \vec{k}_T$. Such replacement must be kept in mind when one expands the hard part in the initial transverse momentum $k_T$.

The above spectrum is proportional to the transverse momentum dependent

gluon distribution. One can interpret the corresponding hard part as the partonic cross section of quark scattering with a *physical* gluon. It vanishes $\widetilde{H}(\vec{k}_T) = 0$ at $\vec{k}_T = 0$, which is quite different from the hard part $H_2^{--}(k_T)$ of quark and longitudinal gluon interaction in the higher-twist approach in a covariant gauge. Therefore, making a collinear expansion of this partonic cross section does NOT correspond to the collinear expansion of the hard part of quark and gluon interaction in the higher-twist approach in the covariant gauge. In order to compare to the results [Eq. (6) or (21)] in the collinear factorized formulation of higher-twist approach, one should integrate the above LCPI result over initial gluon's transverse momentum while keep the transverse momentum $\vec{\ell}_T$ of the emitted gluon fixed.

One can similarly make a small $k_T$ expansion of the hard part of the above LCPI result as in the higher-twist approach,

$$\widetilde{H}(\vec{k}_T) = 4\pi\alpha_s^2 \frac{(\vec{k}_T \cdot \vec{\ell}_T)^2}{\ell_T^6} \left[1 - \cos\frac{\ell_T^2 y_N^-}{2q^- z(1-z)}\right] + \mathcal{O}(k_T^3). \tag{23}$$

After integrating over the initial gluon's transverse momentum and defining the $k_T$-integrated gluon distribution function

$$xG_N(x) = \int dk_T^2 \frac{xdG_N(k_T^2, x)}{dk_T^2}, \tag{24}$$

one can obtain exactly the same gluon spectrum as in Eq. (21) induced by hard-soft scattering in the higher-twist approach. The effect of higher order terms in the above expansion will be suppressed by powers of $\langle k_T^2\rangle/\ell_T^2$ and $\langle k_T^2\rangle\ell_T^2(R_A/q^-)^2$ as in the collinear expansion in the higher-twist approach. Similarly, this approximation is no longer valid for small values of $\ell_T^2 < \langle k_T^2\rangle$.

To study the effect of these higher order terms in the $k_T$ expansion of the LCPI approach, one needs to know the form of $k_T$-dependent gluon distribution function $G_N(k_T^2, x)$. In the GLV approach [9], a static potential model was used for quark medium interaction. The induced gluon spectrum can be written as [25,26]

$$\frac{dN_{\rm GLV}}{dzd\ell_T^2} = \frac{C_A\alpha_s}{\pi^2}\frac{1+(1-z)^2}{z}\int dy_N^-\rho_A(y_N)\int d^2k_T \frac{\sigma_{qN}\mu^2}{(k_T^2+\mu^2)^2} \times \frac{\vec{k}_T\cdot\vec{\ell}_T}{\ell_T^2(\vec{k}_T-\vec{\ell}_T)^2}\left[1-\cos\frac{(\vec{\ell}_T-\vec{k}_T)^2 y_N^-}{2q^- z(1-z)}\right], \tag{25}$$

where $\sigma_{qN}$ is the quark-parton scattering cross section in the medium and $\mu^2$ is the screening mass in the static potential.

If one make a similar small $k_T$ expansion of the hard part in the above GLV result

$$\frac{\vec{k}_T \cdot \vec{\ell}_T}{\ell_T^2 (\vec{k}_T - \vec{\ell}_T)^2} \left[ 1 - \cos \frac{(\vec{\ell}_T - \vec{k}_T)^2 y_N^-}{2q^- z(1-z)} \right]$$
$$= 2 \frac{(\vec{k}_T \cdot \vec{\ell}_T)^2}{\ell_T^6} \left[ 1 - \cos \frac{\ell_T^2 y_N^-}{2q^- z(1-z)} \right] + \mathcal{O}(k_T^3), \tag{26}$$

the induced gluon spectra in Eq. (21) from hard-soft scattering in the higher-twist approach can be recovered if one identifies the soft gluon distribution function as,

$$\frac{2\pi^2 \alpha_s}{N_c} x_\mu G_N(x_\mu) = \sigma_{qN} \mu^2 \log \frac{Q^2}{\mu^2}. \tag{27}$$

The above equation can be rewritten as,

$$\frac{4\pi^2 \alpha_s C_F}{N_c^2 - 1} x_\mu G_N(x_\mu) = \int dk_T^2 \frac{d\sigma_{qN}}{dk_T^2} k_T^2. \tag{28}$$

which relates the soft gluon distribution of the medium and the average transverse momentum weighted cross section in the higher-twist approach [6,23].

Without the small $k_T$ approximation, one can make a variable change $\vec{\ell}'_T = \vec{\ell}_T - \vec{k}_T$ in the integration over $\vec{\ell}'_T$, and complete the integration over the initial gluon transverse momentum $\vec{k}_T$,

$$\frac{dN_{\rm GLV}}{dz} = \frac{C_A \alpha_s}{2\pi} \frac{1 + (1-z)^2}{z} \int dy_N^- \rho_A(y_N) \sigma_{qN} \mu^2$$
$$\times \int \frac{d\ell_T'^2}{\ell_T'^2 (\ell_T'^2 + \mu^2)} \left[ 1 - \cos \frac{\ell_T'^2 y_N^-}{2q^- z(1-z)} \right]. \tag{29}$$

The above result can be compared to the induced gluon spectrum in the higher-twist approach after regularization in $\ell_T^2$, though one should note that $\ell'_T$ after the variable change in the above spectrum is no longer the true transverse momentum of the radiated gluon, which can be integrated over to obtain the transverse momentum integrated gluon spectrum $dN/dz$. The above spectrum can be compared to the collinear factorized form of the spectrum in Eq. (21), if one identifies

$$\frac{2\pi^2 \alpha_s}{N_c} x_\mu G_N(x_\mu) = \sigma_{qN} \mu^2 \tag{30}$$

which differs from Eq. (27) in the small $k_T$ approximation by a logarithmic factor

$$\mu^2 \to \frac{1}{\sigma_{qN}} \int dq_T^2 \frac{d\sigma_{qN}}{dq_T^2} q_T^2 \approx \mu^2 \log \frac{Q^2}{\mu^2}. \tag{31}$$

Therefore, one can consider the above logarithmic factor as the conseqeunce of corrections beyond the small $k_T$ approximation within the static model of the GLV approach.

## 5 Summary

We have briefly reviewed in this paper the framework of twist expansion for the calculation of higher-twist contributions to semi-inclusive cross section of DIS off a nucleus target. We also illustrated the simple partonic picture of the higher-twist result for the induced gluon spectrum and demonstrated its equivalence to the quark-gluon scattering cross section in the collinear factorized framework. We pointed out AZZ's misunderstanding of the collinear expansion in this framework and their confusion over the procedure and region of validity of the collinear approximation. We showed that the small $k_T$ expansion in the LCPI approach without the misconstrued variable change leads to the same semi-inclusive gluon spectrum as in the higher-twist approach. We showed that corrections to the small $k_T$ (or collinear) approximation within the static potential model in the GLV approach lead to a logarithmic factor.

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